Math 250 – Notes: Sect. 4.3 – Riemann Sums and Definite Integrals

Recall from yesterday how we used *rectangles* to estimate the area under a curve.

Picture:

Riemann Sum estimate:

Exact Area:

*Definition of *Definite Integral*:

(Note: f must be continuous for the function to be "integrable.")

-example- Evaluate $\int_{-2}^{1} (x+3)dx$

-example- Evaluate $\int_{-1}^{3} (2x-1)dx$

*The integral does not represent *total area*. The integral represents ____________________________.

Math 250 – Notes: Sect. 4.3 – Riemann Sums and Definite Integrals

-examples- Use geometry to find the value of each integral. (Draw a Picture)

1. $\int_0^5 2dx$

2. $\int_1^3 (x-4)dx$

3. $\int_{-3}^2 |x|\,dx$

4. $\int_0^4 \sqrt{16-x^2}\,dx$

*Properties of Integrals:

1. $\int_a^a f(x)dx =$ ______________

2. $\int_a^b f(x)dx = -\int_b^a f(x)dx$

3. $\int_a^b k\, f(x)dx =$ ______________________

4. $\int_a^b [f(x) \pm g(x)]dx =$ ___________________________

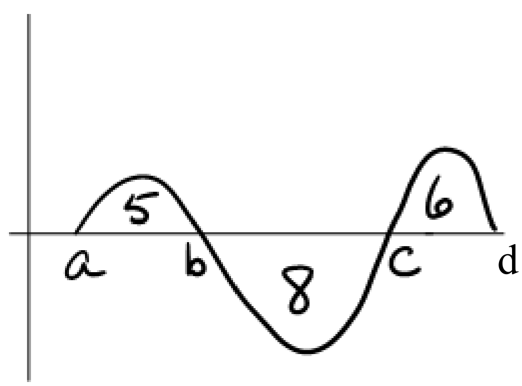
5. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

-example- GIVEN: $\int_a^b f(x)dx = 5,\ \int_a^b g(x)dx = -3$, find

1. $\int_a^b [2f(x) + g(x)]dx$

2. $\int_b^a [f(x) - g(x)]dx$

3. The picture shows the AREAS of the shaded regions.
Find the value of each integral.



a. $\int_a^b f(x)dx$

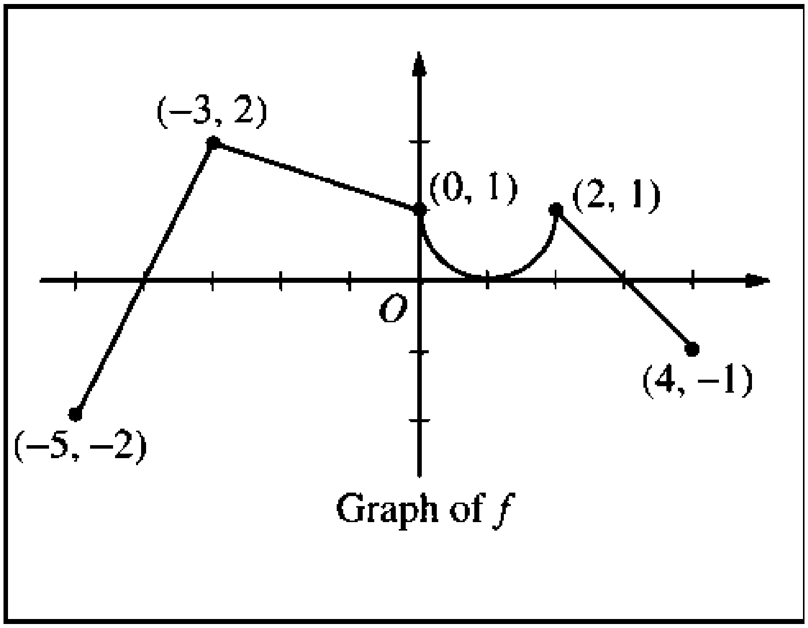
b. $\int_b^c f(x)dx$

c. $\int_a^c f(x)dx$

d. $\int_a^d f(x)dx$

e. $\int_d^a f(x)dx$

4.



The graph of the function, *f*, consists of a semi-circle and three line segments.

Let *g* be the function given by $g(x) = \int_0^x f(t)dt$. Find each of the following:

a. $g(0)$

b. $g(2)$

c. $g(4)$

d. $g(-5)$